Physics 302 Photonics
Homework 3 - SOLUTIONS

(3.49) (a) $K_E(\text{ice}) \approx 1$, $K_E(\text{water}) \approx 80$ for microwaves. Explain

We know: $n \approx \sqrt{K_E}$

and $$n^2 = 1 + \frac{Nq_e^2}{\epsilon_0 m_e}\left(\frac{1}{\omega_0^2 - \omega^2}\right)$$

Consider what the resonance frequencies, ω_0, are for ice versus liquid water, namely which substance has larger ω_0?

- Structure of water: contains strong hydrogen bonds within molecules, much weaker bonds between molecules
- Structure of ice: contains strong bonding within this tetrahedral structure

For "restoring" force constant, k (related to the bonding strength)

$$\omega_0 = \sqrt{k/m}$$

$\Rightarrow$ Substances with large k will have large ω_0.

$\Rightarrow$ $\omega_0(\text{ice}) > \omega_0(\text{liq. water})$

But $\omega(\text{microwave}) \lesssim \omega_0(\text{liq. water}) \ll \omega_0(\text{ice})$

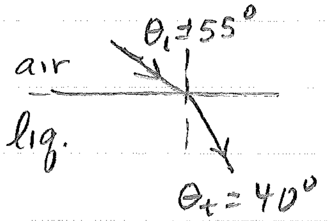
$$\Rightarrow n(\text{liq water}) = 1 + \text{const}/(\omega_0^2 - \omega^2)$$

will be much larger than $n(\text{ice})$ since $\omega(\text{microwave})$ is closer to the resonant frequency of liquid water than that of ice.

(4.15) $\lambda_0 = 600\ nm$

$$\lambda = \frac{\lambda_0}{n} = \frac{600\,nm}{1.5} = \boxed{400\,nm = \lambda} \text{ or } \boxed{\text{violet}}$$

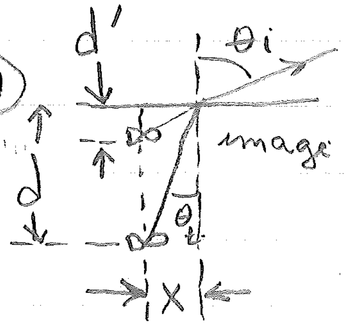
(4.16)



$$n_{air} \sin 55^\circ = n_{liq} \sin 40^\circ$$

$$n_{liq} = \frac{1 \sin 55^\circ}{\sin 40^\circ} = \boxed{1.27 = n_{liquid}}$$

(4.21)



$$\frac{x}{d} = \tan\theta_t$$

$$\frac{x}{d'} = \tan\theta_i$$

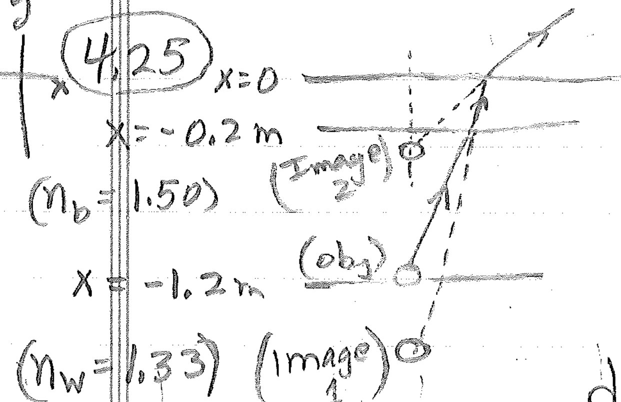
$$\frac{d'}{d} = \frac{\tan\theta_t}{\tan\theta_i} \approx \frac{\sin\theta_t}{\sin\theta_i} \quad \text{for small angles}$$

or $\dfrac{d'}{d} \cong \dfrac{n_i}{n_t}$. For $\theta_i = 0$, $\dfrac{d'}{d} = \dfrac{n_i}{n_t}$

But $n_i = 1$, $n_t = \frac{4}{3}$

$$\boxed{\frac{d'}{d} = \frac{1}{4/3} = \frac{3}{4}}$$

4.25



Note: rays are really normal, drawing for calculation purposes only.

$$d' = d\,\frac{n_i}{n_t}$$

(a) Start from bottom, $n_t = 1.33$, $n_i = 1.50$

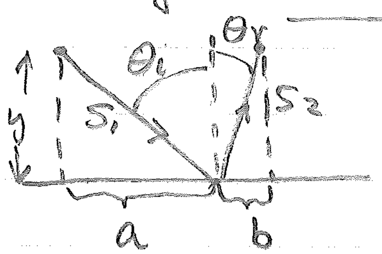
$$d' = (1\,m)\,\frac{1.5}{1.33} = 1.13\,m \text{ (below water/benzene layer)}$$

(b) Use that image as the object for the next interface, air/benzene:

$$d'' = (1.13 + 0.2)\,m\left(\frac{1.0}{1.50}\right) = 0.89\,m$$

$\Rightarrow$ image is 0.89 m below air/benzene interface

4.32



$$t = t_1 + t_2 = \frac{S_1}{v} + \frac{S_2}{v}$$

$$= \frac{1}{v}(S_1 + S_2)$$

Minimizing t is equivalent to mimizing $vt = S_1 + S_2$.
let $a + b = c = \text{const}$

$$S = S_1 + S_2 = \sqrt{y^2 + a^2} + \sqrt{y^2 + b^2}$$

$$= \sqrt{y^2 + a^2} + \sqrt{y^2 + (c-a)^2}$$

Use a as the variation parameter and take derivative

Then we solve the following:

(4.32) Cont'd $$\left.\frac{\partial s}{\partial a}\right)_{\substack{c=\text{const}\\ y=\text{const}}} \overset{\text{set}}{=} 0$$

ie, $$\frac{2a}{\sqrt{y^2+a^2}} - \frac{2(c-a)}{\sqrt{y^2+(c-a)^2}} = 0$$

This means $$2a\sqrt{y^2+(c-a)^2} - 2(c-a)\sqrt{y^2+a^2} = 0$$

Move 2nd term to RHS and square both sides after dividing by 2:

$$a^2[y^2+(c-a)^2] = (c-a)^2(y^2+a^2)$$

These cancel

$$a^2y^2 = y^2(c-a)^2$$

$$a^2 = (c-a)^2$$

$$= c^2 - 2ca + a^2$$

$$\Rightarrow \quad c-2a=0 \Rightarrow a = c/2 \Rightarrow b = c/2$$

that is, $s_1 = s_2 \Rightarrow \frac{y}{s_1} = \frac{y}{s_2}$

or $\sin\theta_i = \sin\theta_r$

or $\boxed{\theta_i = \theta_r}$